3A & 3B Mathematics

Test 1 2009

50 marks

50 minutes + 2 minutes reading

Name SOLUTIONS

1. [2, 1, 2, 2 marks]

A winery produces a fine liquer which is predicted to increase in value by 4·8% p.a.

(a) How much will a bottle of liquer be worth in 2 years time, if it presently sells for $45?

$A = P(1+i)^n$

$= 45(1+0.048)^2 = 49.42$

(b) How much will a bottle of liquer be worth in n years time, if it presently sells for $45?

$A = 45(1.048)^n$

(c) The winemaker plans to release the liquer when it reaches a value of $100. How long will they have to wait for this to be the case?

100

$n = 17$ ✓

(d) Another red wine produced by the winery is increasing in value at 2·3% p.a. If a large flagon presently sells for $74, how long will it be before the Liquer becomes more expensive than the flagon?

$45(1.048)^n > 74(1.023)^n$

22 ✓

Sequences

$a_{n+1} = a_n \times 1.048 \times 1$

$a_0 = 45$

$b_{n+1} = b_n \times 1.023 \times 1$

$b_0 = 74$

2. [2, 1, 2, 2 marks]

Given p(x) = 5x + 3 and q(x) = 2 - x, find the following:

(a) the point of intersection of the two lines,

$5x - 3 = 2 - x$

$6x = 5$

$x = 5/6$

2D

▨

☐

$\left(\frac{5}{6}, \frac{7}{6}\right)$ ✓

(b) p(4) $= 23$

(c) p(q(-1))

$q(-1) = 2 - (-1)$

$= 3$

$p(3) = 5(3) + 3 = 18$ ✓

(d) the value of k for which p(k) = - 2.

$5k + 3 = -2$

$5k = -5$

$k = -1$

3. [1, 1, 3 marks]

A function has a defining rule $y = 2x^2$

Determine the defining rule for the new function if the graph of this function is

(i) moved 2 units left, $y = 2(x+4)^2$

(ii) reflected in the y-axis, $y = 2(-x)^2$

(iii) reflected in the x-axis, then moved 3 units right and then 1 unit up.

$$y = -2(x-3)^2 + 1$$

4. [4 marks]

A cubic polynomial intersects the x-axis at x = -2, 3, 5.

Given that the graph goes through the point (4, 2) find the equation for the polynomial in the form, $y = ax^3 + bx^2 + cx + d$.

$(-2, 0)$

$(3, 0)$

$(5, 0)$

$(4, 2)$

$$y = -\frac{1}{3}x^3 + 2x^2 + \frac{1}{3}x - 10$$ < Starts Cubic eq

At (4, 2)

$$y = a(x+2)(x-3)(x-5)$$
$$2 = a(4+2)(4-3)(4-5)$$
$$2 = a(6)(1)(-1)$$
$$2 = -6a$$
$$a = -\frac{2}{6}$$
$$= -\frac{1}{3}$$

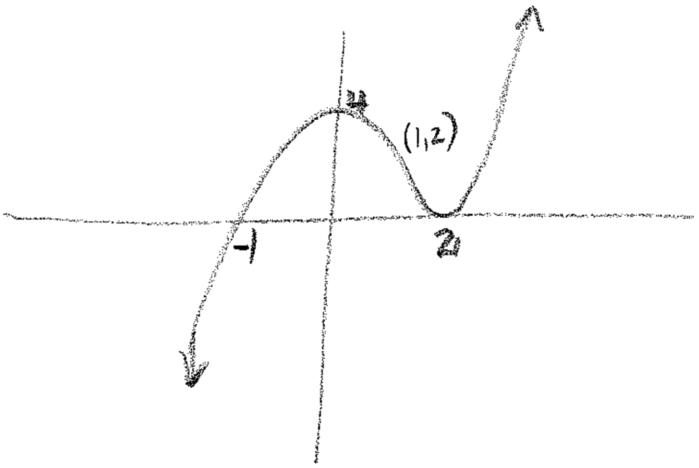
$$y = -\frac{1}{3}(x+2)(x-3)(x-5)$$
$$= -\frac{1}{3}(x^2-x-6)(x-5)$$

5. [6 marks]

With the aid of a graphic calculator produce a sketch of

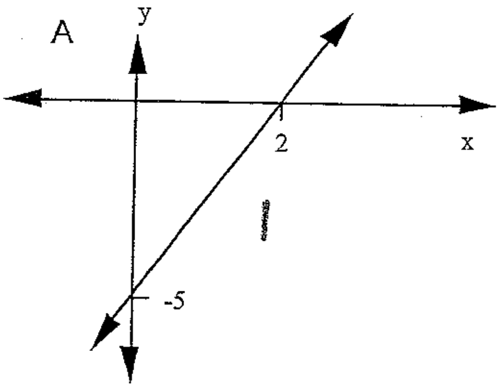
$$y = x^3 - 3x^2 + 4$$

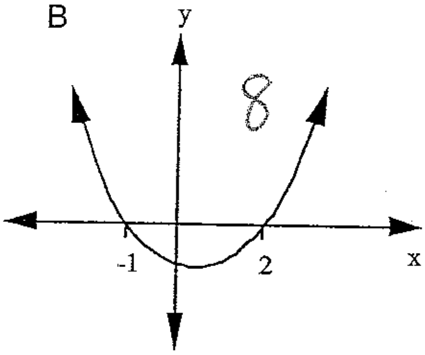
Indicate any turning points , intercepts with the axes and points of inflection. If any rounding is necessary give answers correct to 2 decimal places.

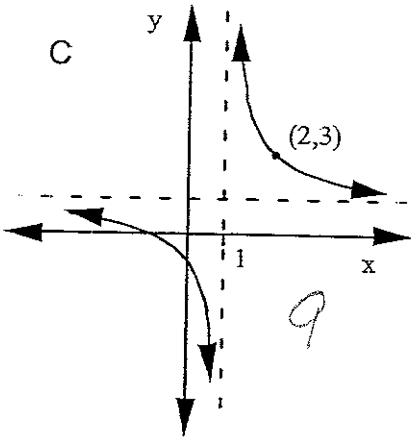


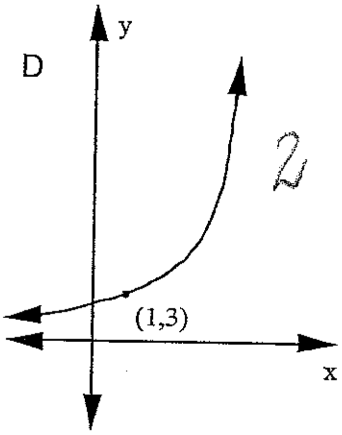
6. [2, 2, 2, 2, 2 marks]

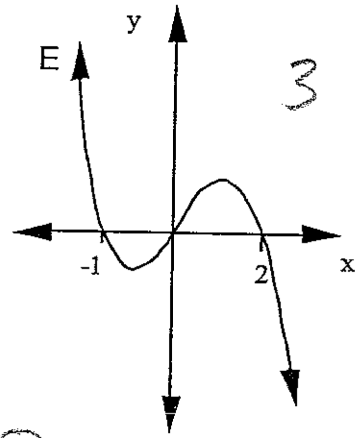
Match each of the graphs below with its corresponding function.
Choose from the functions listed below, where a, b, c, d and e are positive integers:











[✓ 1.] $y = ax - b$ ✓ [✓ 2.] $y = d^x + 1$ [✓ 3.] $y = -ax^3 + x^2 + dx$

~~4.~~ $y = x^3 - ax - b$ ~~5.~~ $y = c^x - 1$ 6. $y = \frac{1}{x+c}$

~~7.~~ $y = x^2 + x - e$ [✓ 8.] $y = x^2 - x - d$ ✓ [✓ 9.] $y = \frac{1}{x-b} + a$

~~10.~~ $y + a\cancel{x} = b$ ~~11.~~ $y = \frac{1}{x-a}$ ~~12.~~ $y = x \cancel{+c}$

$y = -ax + b$

$y = \frac{1}{x-1} + 1$

$y = (x+1)(x-2)$
$= x^2 - x - 2$

7. [2, 3, 3 marks]

State the domain and range for the following functions:

(a) {(2, 3), (1, -9), (0, 4), (-3, 4), (-2, 5), (6, 1)}

$x = \{2, 1, 0, -3, -2, 6\}$

$y = \{3, -9, 4, 5, 1\}$

(b) $y = x^2 + 4x + 3$

$x \in \mathbb{R}$

$y \geq -1$

$\frac{-b}{2a} = \frac{-4}{2} = -2$

$y = 4 - 8 + 3$

$= -1$

(c) $y = \frac{1}{2x - 3} + 1$

$x \in \mathbb{R}$ $x \neq \frac{3}{2}$

$y \in \mathbb{R}$, $y \neq +1$

8. [3 marks]

Given the graphs for $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = ex^2 + fx + g$, for real constants a, b, …,g, solve to 1 decimal place, the equation $f(x) = g(x)$.

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decompressor
are needed to see this picture.